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We can not assume men to be wholly blind to the distinctions that they are compelled to draw every day, and some of which they can only draw correctly after giving much thought and pains to the matter. It remains for the philosopher to sift their statements, to weigh the justice of each, and to look into this question of their mutual consistency. I shall turn to the philosopher in the next paper.

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STUDIES IN THE STRUCTURE OF SYSTEMS

4. THE GENERATING PROBLEM[1]

THE fight for postulates and against axioms is a fight for freedom in mathematics and science. It is this aspect which thus far has been dominant in our discussion. Unlimited possibilities are therewith opened up; by surrendering the idea of "self-evidence" as a necessary requirement which the starting-point of "deduction" must satisfy, mathematicians have not only deepened their insight into the real relations between the propositions of a system, not only perfected their initial sets, not only increased the rigor of their deductions: the real logical requirements which a deductive system should satisfy to be acceptable have come to light;[2] for only if many accounts are possible can there be selection.

So far as the actual work in mathematics is concerned, nothing further, it would seem, need be added to what has already been said on this point. But for a theoretical account of the deductive system form it is insufficient merely to speak of "postulates" and the "postulate method." This theoretical insufficiency will become apparent whenever the method is applied to fields that have not already been worked over frequently; and it evinces itself in certain puzzling questions which sometimes trouble even the mathematician. That we can, to a degree not yet determined, but apparently unlimited, interchange theorems and postulates, *i. e.*, that the distinction between postulates and theorems is not inherent, not absolute, but relative to the order in a particular system, all this must be maintained. But, if the postulates appear as merely assumed, are they, therefore, purely *arbitrary?* And if they have no warrant of their own, how can a deduction from them add anything to the validity of a proposition? What is the real function of these postulates?

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[1] Some of the material of this paper was presented to the American Philosophical Association at its meeting at Yale University, in January, 1910. In the present paper I limit myself to a consideration of the "generating problem" in a *deductive* system.

[2] *Cf.* my paper, "Critique of Cognition and its Principles," this JOURNAL, Vol. VI., page 281.

Some at least of these questions demand an answer in a study of the structure of systems; we shall revert to them in the examination of the nature of *proof* in a deductive system. The difficulty for which the present paper shall try to find a remedy may be stated somewhat as follows. When we establish a set of postulates for "Euclidean plane geometry" we can not use propositions of "algebra." But if postulates are merely "assumed," *why can we not assume anything whatever and develop a system?* Of course, the concept of "truth" enters here, and in so far our question must be answered in "critique of cognition"; but quite apart from the question of "truth," *we require a principle which will determine that certain propositions, true or otherwise, can not by any possibility be amongst the particular set of postulates.* And if we speak of postulates merely as propositions from which the system starts, this question is left open. We need a principle which will demarcate the field in which the "postulates" are assumed.

It is easy to see how so important a question could have been disregarded in the work of mathematicians. They were never trying to establish a set of postulates as such, but a set of postulates of "algebra," or "geometry," etc. *The field was understood as being already well determined,* and the set of postulates determined in reference to it. *But what did determine the field?*

Such questions lead us to conceive the deductive system form in a new way. Thus far we have had in mind propositions in relation to other propositions, forming a system in their relationship. But now we come to conceive of propositions as related, not merely to other propositions, such as the postulates, but to a distinct class of logical entities which may be called *"problems."* We conceive every proposition, and every system of propositions, as definitely related to a "problem" of which it is the *"solution."* And this relation of a proposition to its problem is essential not only for the logical meaning and bearing of the proposition, and not only for the determination and limitation of its "truth," but also for a complete account of the *structure* of a system. It is this structural aspect of the concept of "problem," restricted to *deductive* systems, which shall occupy us for the present.

By "problem" I do not mean a mere "question," not an expression of "ignorance," or "wonder," or "desire to know." All these are expressions for *subject-relations.* They indicate problems which we meet, or set ourselves. The term "problem" shall here designate a logical entity distinct from the subject-relation into which it may enter to me or to you. I may "set" myself a problem, I may "acknowledge" a problem, and thus enter into subject-relation to it; however vital this relation may be for me or for the problem, what a problem *is* is distinct from *my relation to it.*

This comes out clearly when we treat so-called "impossible" problems. A problem may be "impossible" for me in that I can not find a solution for it. But this psychological impossibility does not constitute a logical impossibility, even if it is extended so as to include *all* (present, past, or future) human beings; *mere extension of a "psychological" relation does not make it "logical."* Numerous problems in mathematics are still (psychologically) "impossible," though in the case of many we can prove that they are "logically" possible. That, on the other hand, a logically "impossible" problem may be psychologically possible the French Academy experiences every year many times when new, and new "solutions" of such problems as the "quadrature of the circle" are presented to it by hopeful aspirants to immortality. That these "solutions" are "logically" incorrect deprives them apparently little of their "psychological" correctness, clearness, evidence, certainty.

But if we do not define "problems" in terms of our "ignorance," how shall we define this logical entity to which a proposition is said to be related? I shall attempt to answer this question in terms of *relation,* and my procedure will be an analysis of what is meant by "problem" in mathematics. It may, however, be well to state that this "analysis" is, at bottom, a "construction": it is the construction of a solution to a particular problem, namely, what a problem is; and this solution is to be submitted, so far as its truth is concerned, to the criteria of critique of cognition.

Problems are "felt" in various degrees of clearness, and as such may control our actions, be it in the practical concerns of every-day life, be it in purely theoretical considerations. The mathematician, working through a particular field of his science, may "feel" that something is wrong, may "perceive" that a certain proof makes tacit assumptions, and this "feeling," this "perceiving" may direct him to search for the assumption, to correct the error. But problems are amenable to logical investigation only when they have been *stated,* however imperfectly. At all events, we shall deal here with problems only in so far as they are stated; and our question is: how are statements which represent problems distinct from those which represent solutions?

If I state the problem: what is the shortest distance between two points in the Euclidean plane? I answer: the straight line. What have I done? The "what" merely fills a blank; but it indicates the *locus of a concept,* namely, straight line. I might have put the problem in the form: "x is the shortest distance between two points," if I had made the agreement that the letter x shall indicate the locus of a concept. The form of a "question" is therefore merely a grammatical device; and our ready understanding of this device covers up an essential feature of the problem-statement, namely, that it is in itself *incomplete:* it does not "propose" anything, it does not "assert" anything; *it requires another statement for its completion.* But it states a relation between concepts by which another concept is determined.

This can be analyzed still further, and another example may be more serviceable: Which is the larger of two chords in a circle? The definite relation, namely, between the sizes of two chords in a circle; x is the one nearer the center. This statement of the problem is very imperfect; what it is meant to bring out is this: the problem states a definite relation, namely, between the sizes of two chords in a circle; but whilst the statement: chord a is larger than chord b would be complete in itself, *i. e.,* a proposition, the addition of the x, which transforms this proposition into a problem, leaves the whole statement incomplete; it becomes a proposition again only after the solution is added; and this is done by introducing a new relation, namely, "distance from the center," and asserting that these two relations "size of chord" and "distance from center" belong together. *The problem is this demand for the establishment of a relation between an initial relation and a new relation "determined" by it.* And this seems to be essentially the logical character of any problem.

Instead of speaking of an initial relation it would be more correct to speak of a *system* of (one or more) initial relations which the problem states. For even in the example given we have not only the relation "a is greater than b," but also "a and b are chords in a circle," *i. e.,* straight lines in definite relation to a circle. These relations are explicitly stated or indicated; others are merely implied. Thus it is implied that the stated relations obtain in a "Euclidean plane," which expression is itself merely an indication of relations explicitly stated in the "postulates." And it is impossible to disregard these implied relations without completely changing the problem. The importance of these implied relations makes it imperative that somewhere they ought to be explicitly stated and that the reference to them be unequivocal. The logical tendency of the whole work in modern mathematics, and the inspiration of so-called mathematical logic, is to make explicit the implicit assumptions. This is at bottom what is meant by the slogan, opposition to intuition.

Let us designate the system of relations explicitly or implicitly stated in a problem by R_P, and the relations introduced by the solution R_S, then we may write

$$R_S = F(R_P),$$

i. e., R_S is a function of R_P. We shall have to examine this functional relation between R_S and R_P in a later paper; here it is important to state at least one of its properties: it is not necessarily a one-one relation, *i. e.*, given a system R_P, a system R_S is not necessarily uniquely determined, even if the problem is "well determined." I call a problem "well determined" if of every system of relations it is determined whether it is a solution of the problem or not. Well-determined problems are, then, of two classes, which may be called (1) *uniquely determining problems,* or *problems with uniform solutions,* and (2) *multiply determining problems,* or *problems with multiform solutions.* In the first case, one and only one solution belongs to the problem; in the second, the problem has two or more solutions. And the important question arises: provided a problem is well determined, can it always be transformed into a uniquely determining problem? And what requirements must a uniquely determining problem satisfy?

The recognition that there are problems with multiform solutions is of very great importance, particularly as the naïve mind, owing to a lack of fertility, is inclined to believe that any problem presented, if it is well determined, must be uniquely determining; and, when one solution is found, holds this to be the *only* possible solution. The strength of our conviction of the truth of a solution is often inversely proportional to our ability to find other, differing solutions. This was true until recently of such problems as the establishment of a set of postulates of geometry, of algebra, or of principles of mechanics; and nothing seems more startling in the outcome of the work of intrepid mathematicians than the conviction that most problems of a fundamental nature are multiply determining.

Many problems are multiply determining only improperly; they can be transformed into problems with uniform solutions by slight modifications in the statement of the problem. But even in the case of proper multiply determining problems, a transformation of the problem into a uniquely determining problem is often possible and desirable. This can be done by the addition of limiting conditions, by a restriction of the range of the variables, etc. It is not possible, neither is it necessary, to enter here into a general discussion of the methods by which this transformation can be effected. But it is important to state that all our problems have multiform solutions, or fall short even of the property of being well determined, unless the following two conditions are satisfied: (1) that the *type* of the solution is determined; (2) that the *critical ideal* of the type is defined. The "type" of a system of propositions, stating the logical properties of a system of logical entities, refers to the *form* as distinguished from the logical *content*. Thus the same content can be stated in distinct forms, such as "mere enumeration," or "deductive system form," etc.; it is these classes of forms which I here call "types"; and the solution of a problem may be required to be of the type of the deductive system form, or of the type of mere enumeration, etc. When a problem is stated in mathematics, the type is usually understood to be that of the deductive system form; but when a problem is stated in physics, the type would be doubtful, as differing types are in use there, were it not that the type required is ordinarily sufficiently indicated by the natural surroundings of our problems. Yet it may always be called a structural fault of a problem if the required type of the solution is left to these suggestions and implications.

From the type is to be distinguished the *critical ideal.* The type is merely the logical form; the critical ideal is the standard by which the logical value of a system is determined. It is necessary that the critical ideal of each type be defined. It is possible that this critical ideal is the same for different types, though it is not necessary that this should be so. For the type of the deductive system form, the critical ideal is now emerging, owing to the persistent work at the logical foundations of mathematics, and in my paper on "Critique of Cognition" I have exhibited a set of criteria defining this ideal. Different solutions of this problem are possible and must be judged by their own test, *i. e.,* they must be "self-critical." It may be possible to include in our set defining the critical ideal of the deductive system form criteria (such as "simplicity"), so that the number of possible solutions is reduced; and it may even be possible to transform the problem by this procedure into a uniquely determining problem. These remarks concerning the effect of the critical ideal on the solution of the particular problem of critique of cognition may serve to indicate the general effect of the definition of the critical ideal of a type on the solution of problems belonging to that type.

We conceived that underlying each proposition is a problem to which it belongs and of which it is the solution; and we saw that this "belonging" is not necessarily a one-one correspondence, in the sense that, given a problem, there may exist more than one solution.

The inverse of this is also true: given a proposition, it may be the solution of more than one problem. Whilst nothing prevents our starting from a proposition and seeking the problem, or problems, of which it is a possible solution, it is more important, and in better harmony with the historical development of science as well as with human nature, to start with *the problem as the primary,* and proceed from it to the propositions which are its solution. We introduce therewith a directional element whose importance will become apparent in later papers. From this point of view we consider a proposition as being a response, so to speak, to a problem; as living its logical life merely in so far as it is a solution; as being *"generated"* by a problem; and problems, on account of this call for definite propositions as their solutions, may be said to generate these propositions. The term "generating" would then merely express the logical demand for a solution, which we found to be one of the characteristics of the idea "problem." However, it seems convenient to restrict the use of the term "generating" problem somewhat arbitrarily, so as to avoid a mere pleonasm. In the sense specified above, *all* problems are "generating problems"; I shall, however, apply the term merely to a particular class of problems, which, as *"generating problems"* shall be distinguished from the rest, which I shall call *"special problems."* Let me elucidate what I mean by this distinction.

Problems cohere; they may be grouped and classified, in so far as their systems of initial relations contain common parts. The initial relations of a problem may be conceived as *conditions* which make the solution of the problem possible; for, as the solution will change if we change the initial relations, we can say that the *possibility* of the solution is conditioned by these initial relations. If, therefore, several problems contain, amongst their initial relations, a group which is common to all, they form a system whose possibility is conditioned by this common group of relations. This group may itself be conceived as a system of initial relations of a new problem; and we say then that the several problems are *special* problems of the more general problem.

In order to state this more clearly it will be convenient to introduce here the idea of *"realm of a problem."* We arrive at it as follows. A logical realm is determined by a *condition* (or system of conditions); and *vice versa,* to every condition (or system of conditions), belongs a realm in which the condition is *"satisfied"* (provided we include the *"zero realm"* for inconsistent conditions). Now, any proposition is here considered as the solution of a problem, and the initial relations of the problem as conditions; they determine a realm in which the initial relations "hold" (*i. e.*, in which the conditions are "satisfied"); in this realm, then, the proposition is a solution of the problem; *and if the problem is properly stated, the validity of its solution is restricted to the realm of the problem.* If now the realm of a problem P' is part of the realm of problem P, we will say that P' lies within the realm of P, that it belongs to the realm of P, that it is a special problem within the realm of P, or simply *a special problem of* P. If, in addition, the solution of P forms a system in the ordinary sense of the word (such as geometry, mechanics, etc.) we shall call P a *generating problem.* P is the problem which is conceived as "generating" the system; the system is a possible solution of its generating problem P; the "theorems" of the system are possible solutions of special problems within the realm of the generating problem of the system.

Systems are thus taken out of their self-contained isolation; they are not an arbitrary collection of propositions, which, like rocks in a cyclopic wall, form a system by mere conglomeration. Systems are possible solutions of generating problems, and it is the problem which furnishes the organic connection between the various propositions of the system.

It follows from our definition of a generating problem that its distinction from a special problem is not inherent, not absolute. That lies already in the accidental character of what is usually called a "system." Geometry forms a "system"; its problem is therefore called a "generating problem"; but we may also call the generating problem of geometry a special problem within the realm of the generating problem of mathematics. It is quite conceivable that there are special problems which can never be generating problems, and generating problems which can never become special problems; but ordinarily the distinction between special problems and generating problems is merely relative.

Let us apply some of the foregoing remarks to a special case. Suppose the generating problem of mechanics to be given in the form: "to deduce from the properties of material systems which are independent of time, their phenomena which occur in time and their properties which depend on time."[3] We are not concerned here with the question whether this correctly states the generating problem of mechanics, or not. But in this statement are contained important points which in certain respects anticipate the solution: (1) it conceives mechanics as dealing with "material systems"; (2) the initial relations of this particular system of mechanics are "their properties independent of time" ($=A$); (3) the required relations which are to be deduced represent ''their properties which depend on time'' and ''their phenomena which occur in time'' ($=B$).

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[3] H. Hertz, "Principien der Mechanik."

But this means: (1) This statement of the generating problem of mechanics determines the realm of mechanics: it is restricted to ''material systems''; any problem regarding the properties of material systems will form a special problem in the realm of this generating problem and therefore demand a solution which can be ''deduced'' from the initial set; and none but these. (2) It establishes a criterion for the distinction between the ''fundamental'' and the ''derived'' propositions; this lies in their relation to ''time.'' (3) It determines the type of the solution: B is to be ''deduced'' from A; the solution shall therefore be in the deductive system form.

The critical ideal is not stated in the generating problem, and need not be; it assumes that the critical ideal of the type has already been defined. Toward the solution of this urgent problem Hertz himself made an important contribution.[4]

What the ''principles'' ($=A$) are in detail is not stated in the generating problem and need not be stated in the problem itself; neither does it state which ''fundamental concepts'' will be necessary, excepting that ''time,'' ''material system,'' etc., will either be among them or definable in terms of them.

In a similar way any statement of a generating problem anticipates certain aspects of its solution. But which aspects are anticipated can not be stated in general, not even for a special system such as mechanics; for its generating problem can be stated in many different and still essentially equivalent ways.

The preceding discussion of the meaning and implications of the concept ''problem'' has anticipated some of the *functions* of this concept. I shall, in what follows, discuss these mainly so far as they pertain to generating problems, though, after what has been said regarding the relativity of the distinction between generating problems and special problems, what applies to generating problems applies with certain natural modifications to special problems also.

Generating problems are the logical origin of systems. Socrates urged the methodological importance of ''questions,'' using the τὶ ἐστὶ as the instrument for obtaining *definitions; Plato* recognized the ταυμάζειν as the starting-point of all philosophizing; *modern philosophers* emphasize the ''purpose'' which directs our thinking: they all have in mind the *subject-relation* to which the logical entity ''problem'' more or less closely corresponds. Not so easily recognizable but deeply permeating his whole system is the rôle of the

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[4] *Einleitung* to his ''Mechanik.''

"problem" in *Kant's* philosophy. Some aspects of it, such as the designation of the "object" as an "*x*," I shall discuss at some other opportunity; but the problem-character of the "ideas" will come readily to mind; their "regulative" use, as distinct from the "constitutive" use, of the "categories" is a very characteristic function of problems. At closer inspection it will be seen that even the "transcendental method" should lead to the generating problem. In Kant's system its fulcrum is the existence of mathematics and Newton's mechanics. If they had indeed the property of "self-evidence" and consequent "undeniability" which then they were believed to possess, Kant's transcendental method would proceed without inherent difficulties. But if *different* sets of postulates can be, and have been, exhibited for mathematics as well as for mechanics, we can not arrive at a system of categories and principles ("*Grundsätze*") by asking: which are the necessary presuppositions of "experience," unless we assert dogmatically that these categories and principles remain *invariant* in the different selections of postulates; for which we have, at present at least, no warrant, and which Kant did not and could not have had in mind when he presented his transcendental method. In the concept "problem" Kant's method finds its real fulcrum; we do not start with the accomplished "fact" of Newton's mechanics to find its necessary presuppositions, but with the *problem* of mechanics, of which Newton's system is one of an indefinite number of possible solutions. Problems generate systems; *and the "postulates" of a system are the initial relations, are the conditions which make the solution of the problem possible.*

The generating problem is the selective principle which determines the kind of propositions which may be admitted to the set of postulates, and excludes others as by no possibility admissible to the set, in that they do not contribute to the possibility of a solution of the generating problem. The postulates are, therefore, not "arbitrary," or "mere conventions": they are *necessary,* namely, for the solution of their generating problem. Other sets might be chosen, but some must be chosen if a solution of the generating problem is to be accomplished. All possible sets of propositions are therefore grouped into two classes with respect to a generating problem: those that are possible initial relations of the given problem, and those that are not.

The generating problem determines what is "given" for the particular system which is its solution. "Givenness" is never an absolute property of certain propositions, concepts, attributes, relations, or what not. What is "given" in the realm of one generating problem may be not-given in the realm of another. When I develop a system of geometry from a set of postulates, the postulates are the "given," the "theorems" are not-given; they are to be "found," or "proved." But if I state the problem: "to determine a set of postulates from which Euclidean plane geometry may be deduced," the system of propositions which compose what is called "Euclidean plane geometry," or a certain set of postulates which *defines* Euclidean plane geometry, is the "given"; the new set of postulates is the required solution, *i. e.,* not-given. It is a common prejudice of those who have given little attention to these questions of structure, and who ignore the rôle of the concept "problem," to take it that "sensations" are the "given *per se.*" This is incorrect. Sensations may be the given, namely, in the realm of a certain generating problem; in others they are not.

The generating problem determines what is "essential." How much confusion has resulted from the endeavor to establish certain propositions, or qualities, as "essential *per se*"! Our whole system of "virtues" has been obscured by this attempt. What is "essential" to a system is determined by its generating problem; it is either one of its "defining" properties (initial relations), or a "necessary consequence" from these. But the "same" system (*e. g.,* "man") has very different "essential" properties, according to the particular generating problem.

The generating problem determines the realm of the system. This is perhaps its most striking function, though its full importance does not appear until we come to critique of cognition. I can mention here only that the "truth" of a proposition does not attach to the proposition as such, in isolation as it were, but only in so far as it is part of a system, *i. e.,* relative to a definite generating problem. And the "truth" of a proposition does not extend indefinitely far, but only as far as the realm of the generating problem extends. To put this into language familiar to students of the history of philosophy: *Kant's* main undertaking in his "Critique of Pure Reason" was to demonstrate that the "truth" of the principle of causality does not prohibit the "truth" of the principle of freedom, in that the realm of the principle of freedom lies wholly outside the realm in which the principle of causality is valid. Causality is restricted to "experience"; freedom lies outside of "experience." But "experience" designates here merely a generating problem, and what Kant tried to establish is that the generating problem of "experience" is distinct from the generating problem of "ethics." The term "things in themselves," the distinction between "ideas" and "categories," between "regulative principles" and "constitutive principles," are merely details of Kant's special solution of the problem of separating these two generating problems. Kant has indeed sufficiently indicated the rôle of the concept "problem" in his undertaking; but it is obscured by the psychological turn which he gave his solution.

But if the "truth" of a proposition is thus restricted to the realm of its generating problem, *the separation of generating problems becomes of the utmost importance;* and my own tentative procedure in the beginning finds here its structural justification. It may be opportune to call attention here to a procedure in mathematics which is in harmony with this requirement of the separation of generating problems. It is usual, in mathematics, to state propositions with conditions which definitely restrict their validity; *i. e.*, mathematicians take at first a rather narrow but well-defined realm (or, to put it differently, they limit their generating problems) in which propositions are established. Now it *may* happen that the validity of a proposition extends beyond the realm of the generating problem in which it is proved; the mathematician will then proceed to *"extend"* the realm by special methods; *but in no case can the validity of a proposition be taken for granted in the extended realm until it has been established for the extended realm.* This "method of limitation and extension," as it might be called, has proved of the greatest power in mathematics; without it the present rigor of mathematics would have been unattainable; and it is the main cause of the orderly progress in the development of mathematics which struck Kant so forcibly, the real "royal road" which he tried to find for philosophy.

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REVIEWS AND ABSTRACTS OF LITERATURE

Münchener Philosophische Abhandlungen. Theodor Lipps zu seinem Sechzigsten Geburtstag gewidmet von früheren Schulern. Leipzig: Johann Ambrosius Barth. 1912. Pp. iv + 316.

The book consists of ten essays, no two of which are upon the same subject. Philosophy, logic, epistemology, ethics, and psychology are represented, and it is quite impossible to review the book as a whole. A type of mind such as that of Lipps can not help influencing greatly many pupils. This is clearly seen in some of the essays. Others of the papers have followed the doctrines of Husserl. The book will, therefore, be of special interest to those who wish to trace the range of the Lippsian influence, even if the merits of the essays did not warrant their being read. With this in mind, the reviewer decided to present a summary of all the